

Bayesian Networks in Educational Assessment

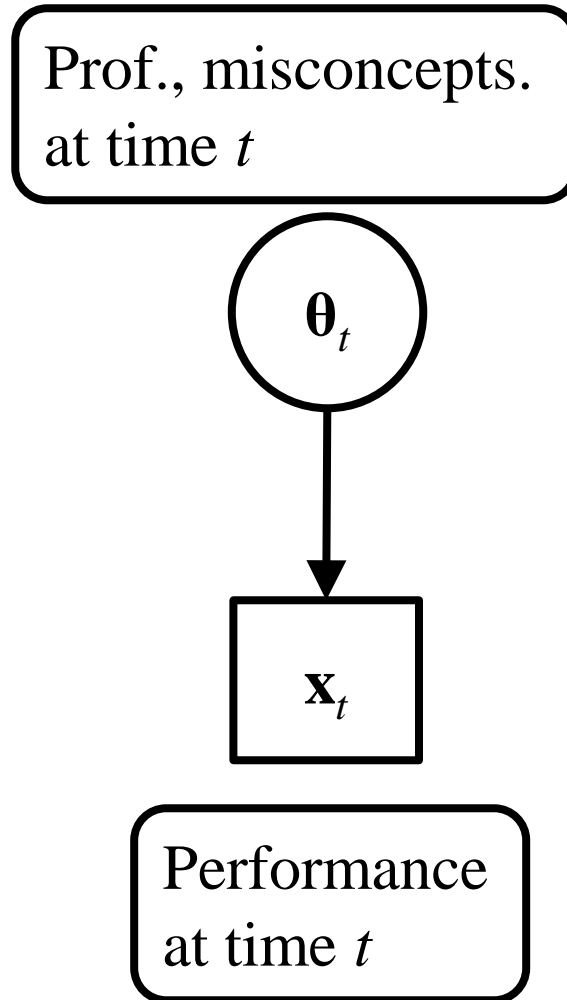
Dynamic Bayesian Networks

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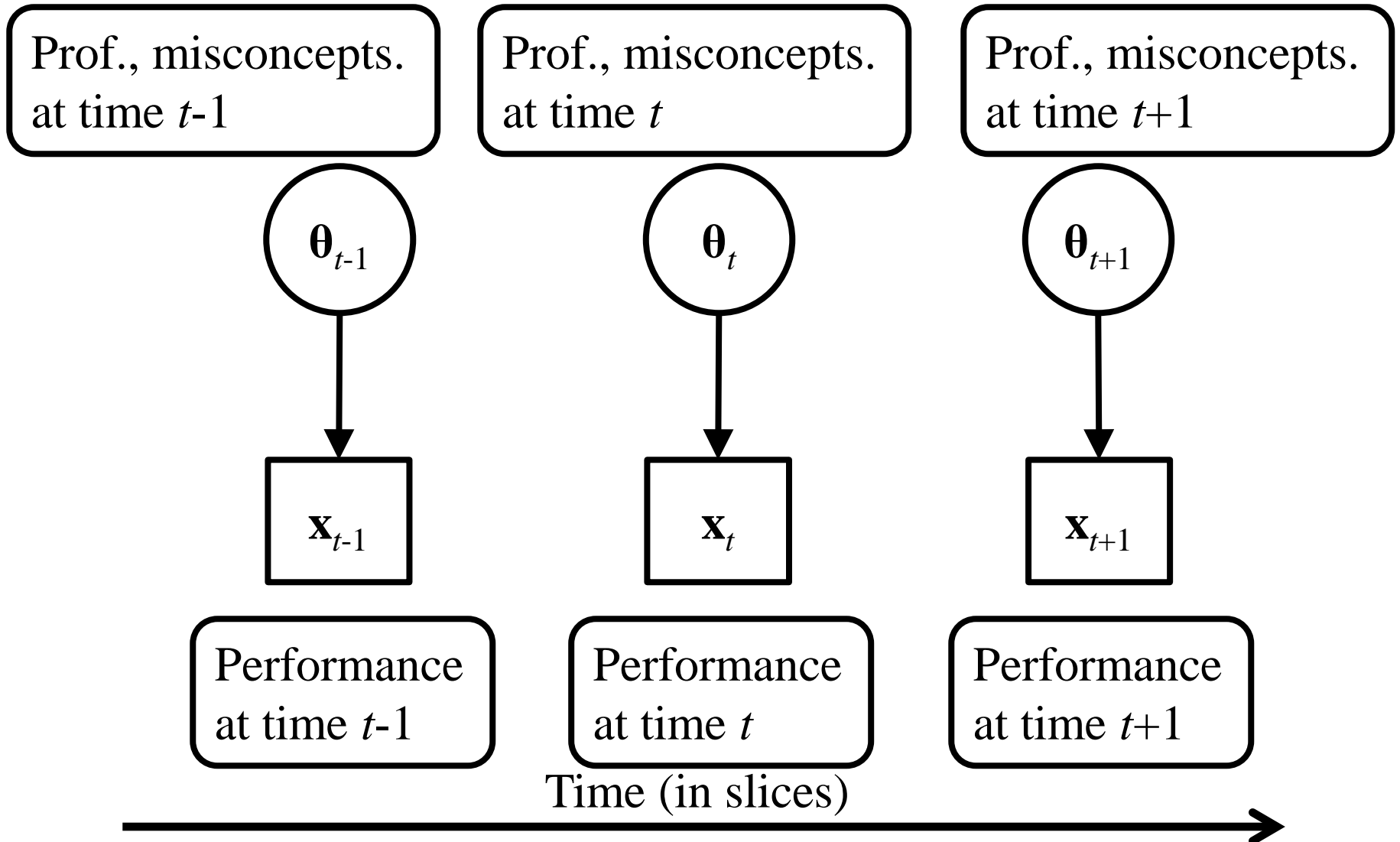
Dynamic Bayesian Networks (DBNs)

- *Dynamic* BNs (DBNs) for modeling longitudinal data
- Bayesian network where variables are repeated, usually over time or related index
- Assessment applications: monitor learning and growth
 - Proficiency and performance on first attempt, on second attempt, etc.
- Knowledge tracing, latent Markov models, latent transition models, growth models

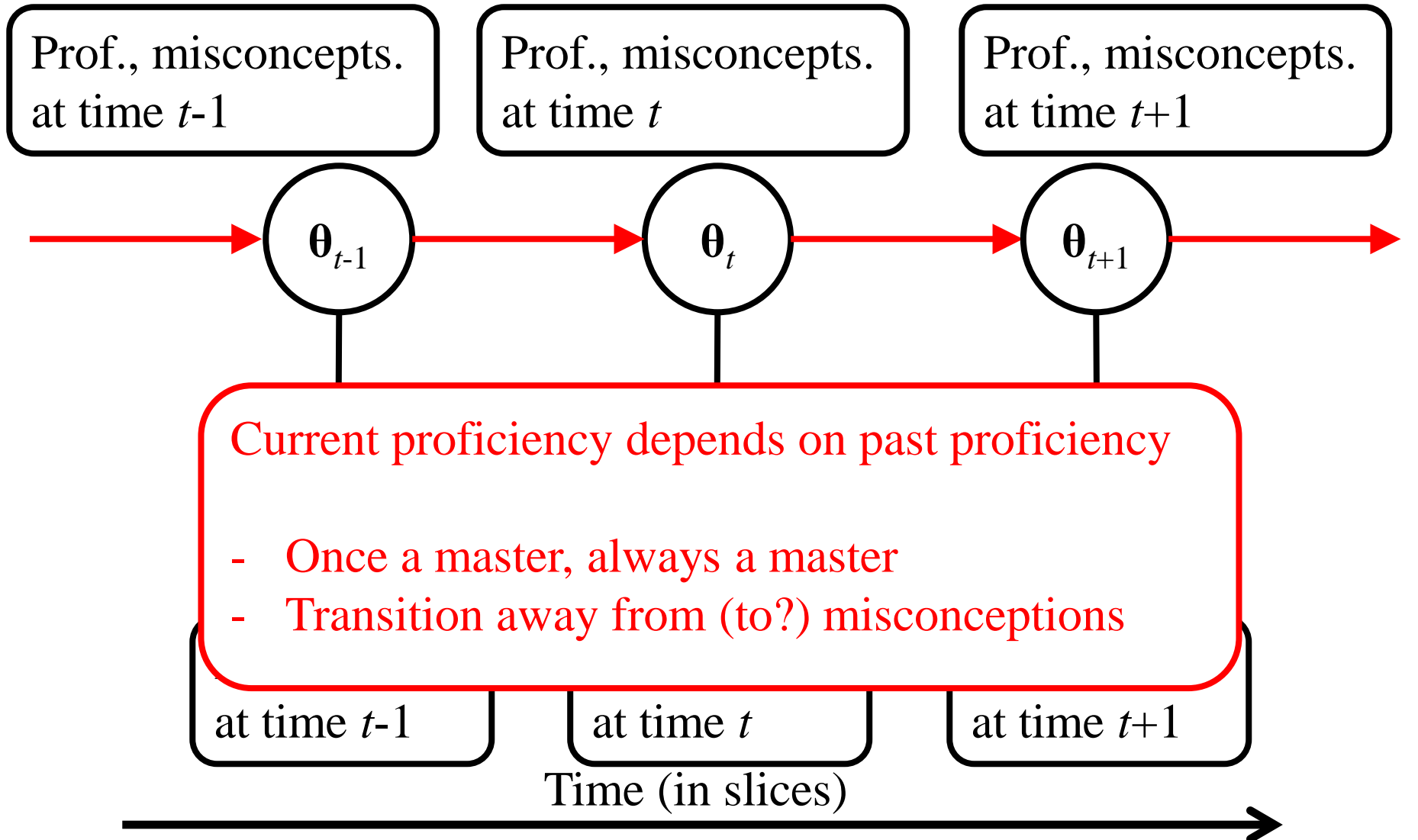
DBN Psychometric Models: Within-Time Component



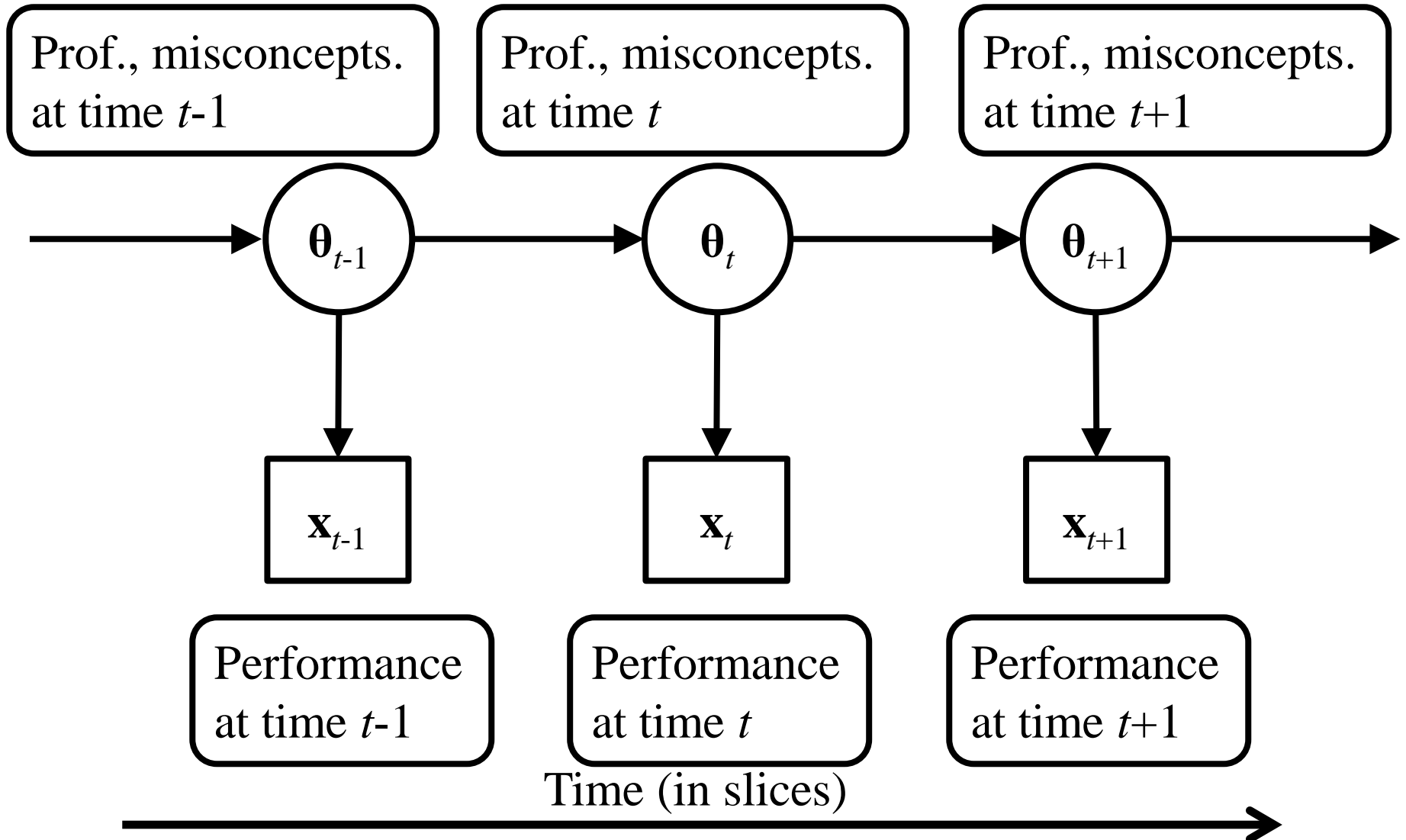
DBN Psychometric Models: Within-Time Component



DBN Psychometric Models: Transition Component

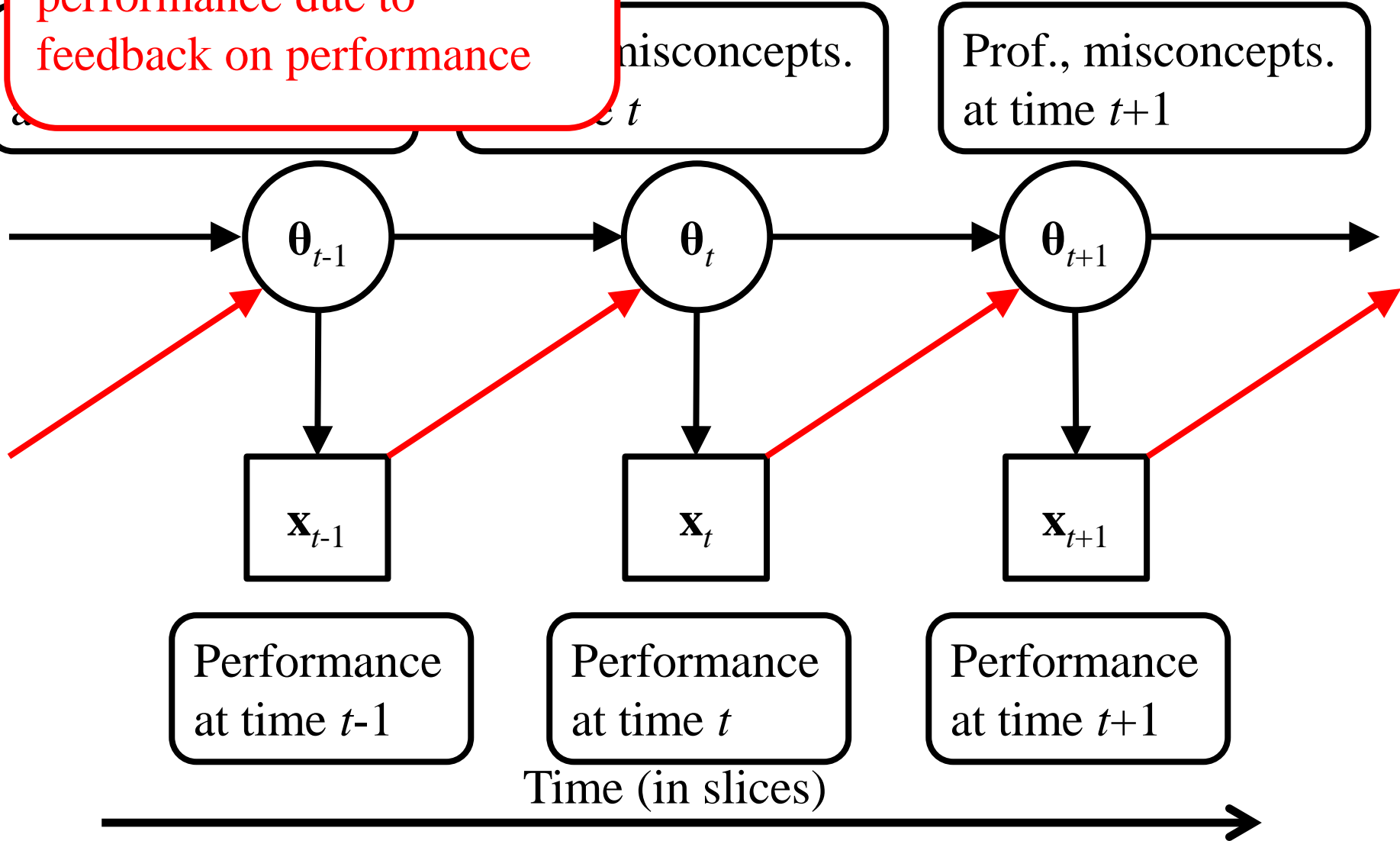


DBN Psychometric Models

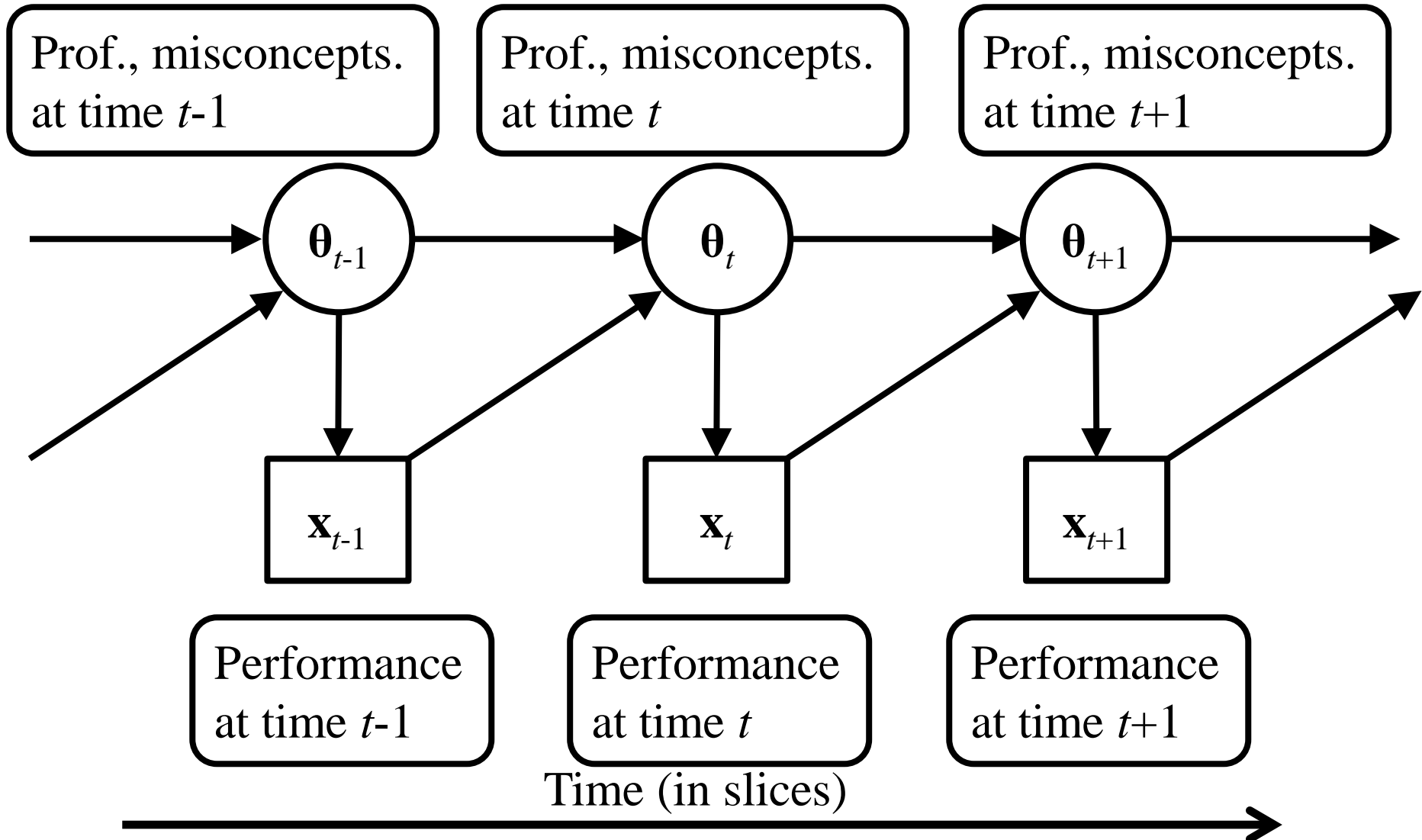


DBN Psychometric Models: Transition Component

Current proficiency depends on past performance due to feedback on performance



DBN Psychometric Models



Example

Example Context: Save Patch

- Educational video game targeting rational number equivalence
 - Adding whole numbers
 - Finding appropriate denominators, fractions < 1
 - Finding appropriate denominators, fractions > 1
 - Adding fractions given correct ingredients
 - Adding fractions greater than 1
- Student lays out ropes for character to navigate across to end
 - Success on a level leads to more complicated levels
 - Advanced levels involve converting ropes (fractions), more complicated layouts, and gaming features (picking up keys, coins)

Example Context: Save Patch

- Complete a level, move on to the next level
- Don't complete a level, try again (and again, and again,...)
- Constructed as a learning tool
- Assesses proficiency of various skills (converting fractions, adding fractions, etc.) and
- Assesses various misconceptions/errors (inclusion, partitioning, etc.)
- Game-playing strategies relevant too (e.g., everything in order)
- Key departures from standard assessment paradigm
 - Feedback (student knows if correctly or incorrectly completed)
 - Learning during assessment (by design!)
 - Performances not conditionally independent (you know what you did, and how it turned out, for the most part)

Dynamic Bayesian Networks (DBNs)

- Characterization of performance
 - Standard solution
 - Alternate solution
 - Incomplete solution
 - Errors (many different kinds)
 - Skipped key
 - Wrong direction
 - Reset solution
- Example: performance on Level 19
 - Assuming the examinee does not have the misconception
 - 2-class latent variable for mastery of whole numbers
 - Probabilities estimated using MCMC, input to Netica
 - Analysis of first four types of performance, attempts resulting in others ignored

Conditional Probability of the Observable

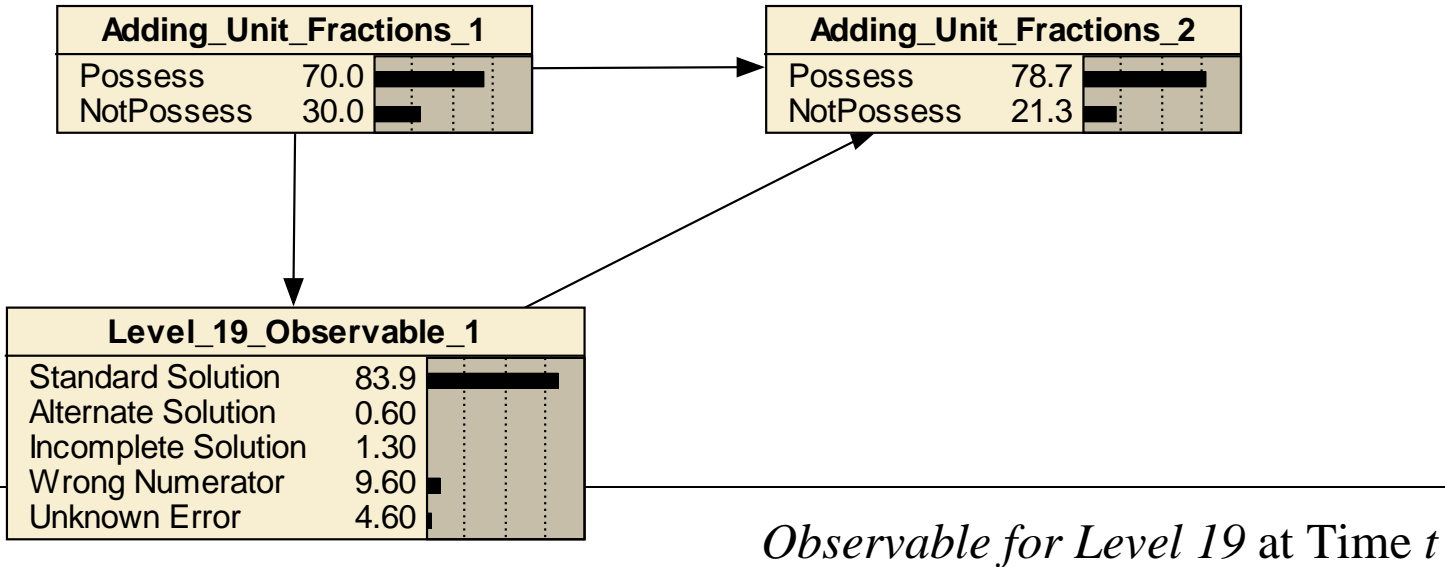
Adding_Unit_Fractions_1		
Possess	70.0	
NotPossess	30.0	

Level_19_Observable_1		
Standard Solution	83.9	
Alternate Solution	0.60	
Incomplete Solution	1.30	
Wrong Numerator	9.60	
Unknown Error	4.60	

$p(\text{Observable for Level 19} \mid \text{Adding Unit Fractions})$

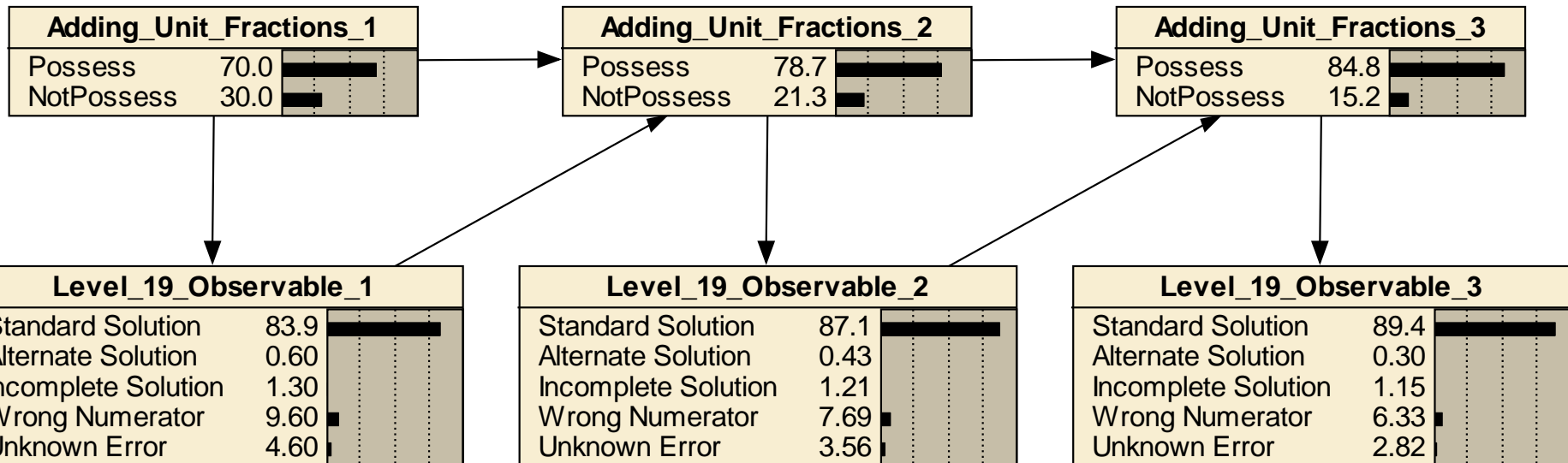
<i>Adding Unit Fractions</i>	Standard Solution	Alternate Solution	Incomplete Solution	Wrong Numerator Error	Unknown Error
Possess	0.95	0.00	0.01	0.03	0.01
Not Possess	0.58	0.02	0.01	0.25	0.13

Transition Probability



<i>Adding Unit Fractions at Time t</i>	Standard Solution	Alternate Solution	Incomplete Solution	Wrong Numerator Error	Unknown Error
Possess	1	1	1	1	1
Not Possess	.38	.17	.19	.20	.09

3 Time Slices



$$p(\text{Possess}) = .70$$

Adding_Unit_Fractions_1	
Possess	70.0
NotPossess	30.0

Adding_Unit_Fractions_2	
Possess	78.7
NotPossess	21.3

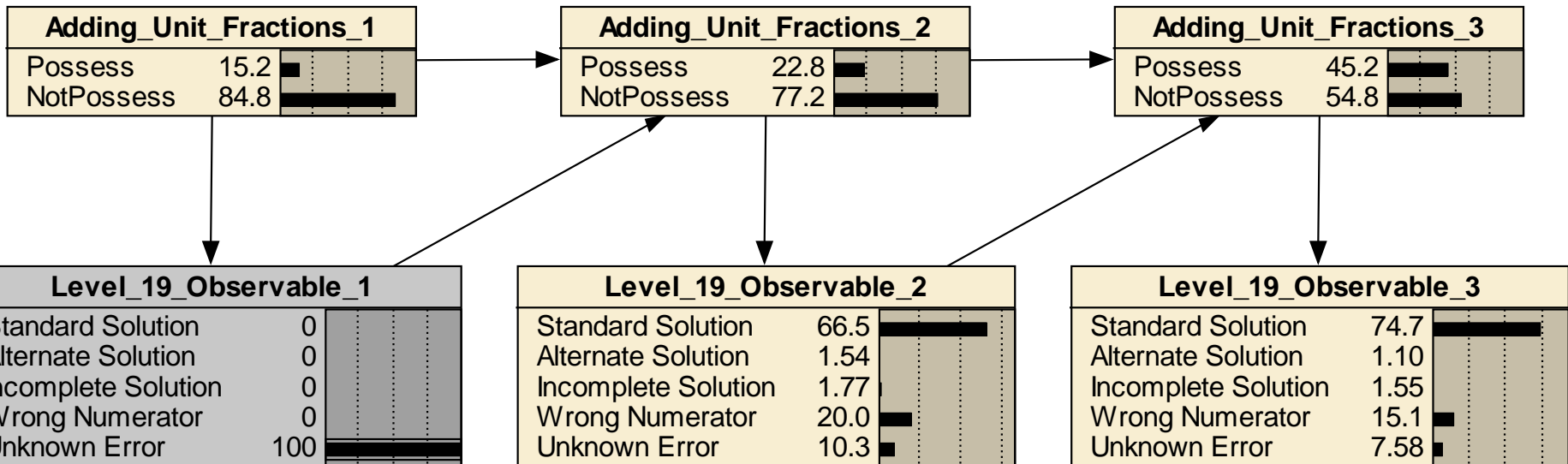
Adding_Unit_Fractions_3	
Possess	84.8
NotPossess	15.2

Level_19_Observable_1	
Standard Solution	83.9
Alternate Solution	0.60
Incomplete Solution	1.30
Wrong Numerator	9.60
Unknown Error	4.60

Level_19_Observable_2	
Standard Solution	87.1
Alternate Solution	0.43
Incomplete Solution	1.21
Wrong Numerator	7.69
Unknown Error	3.56

Level_19_Observable_3	
Standard Solution	89.4
Alternate Solution	0.30
Incomplete Solution	1.15
Wrong Numerator	6.33
Unknown Error	2.82

1st Observation



Observe an unknown error on the first attempt

Posterior Probability

$$p(\text{Possess at time 1} \mid \text{Unknown Error at time 1}) = .15$$

Adding_Unit_Fractions_1	
Possess	15.2
NotPossess	84.8

Adding_Unit_Fractions_2	
Possess	22.8
NotPossess	77.2

Adding_Unit_Fractions_3	
Possess	45.2
NotPossess	54.8

Level_19_Observable_1	
Standard Solution	0
Alternate Solution	0
Incomplete Solution	0
Wrong Numerator	0
Unknown Error	100

Level_19_Observable_2	
Standard Solution	66.5
Alternate Solution	1.54
Incomplete Solution	1.77
Wrong Numerator	20.0
Unknown Error	10.3

Level_19_Observable_3	
Standard Solution	74.7
Alternate Solution	1.10
Incomplete Solution	1.55
Wrong Numerator	15.1
Unknown Error	7.58

Observe an unknown error on the first attempt

Posterior Predictive Probability

$$p(\text{Possess at time 2} \mid \text{Unknown Error at time 1}) = .23$$

Adding_Unit_Fractions_1	
Possess	15.2
NotPossess	84.8

Adding_Unit_Fractions_2	
Possess	22.8
NotPossess	77.2

Adding_Unit_Fractions_3	
Possess	45.2
NotPossess	54.8

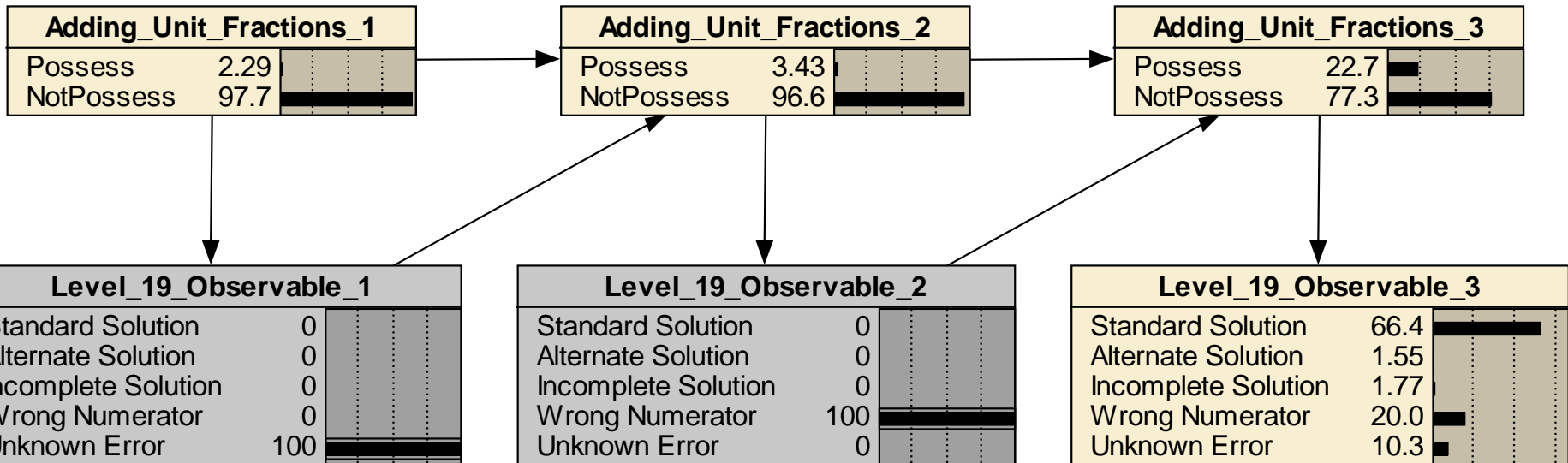
Level_19_Observable_1	
Standard Solution	0
Alternate Solution	0
Incomplete Solution	0
Wrong Numerator	0
Unknown Error	100

Level_19_Observable_2	
Standard Solution	66.5
Alternate Solution	1.54
Incomplete Solution	1.77
Wrong Numerator	20.0
Unknown Error	10.3

Level_19_Observable_3	
Standard Solution	74.7
Alternate Solution	1.10
Incomplete Solution	1.55
Wrong Numerator	15.1
Unknown Error	7.58

Observe an unknown error on the first attempt

2nd Observation

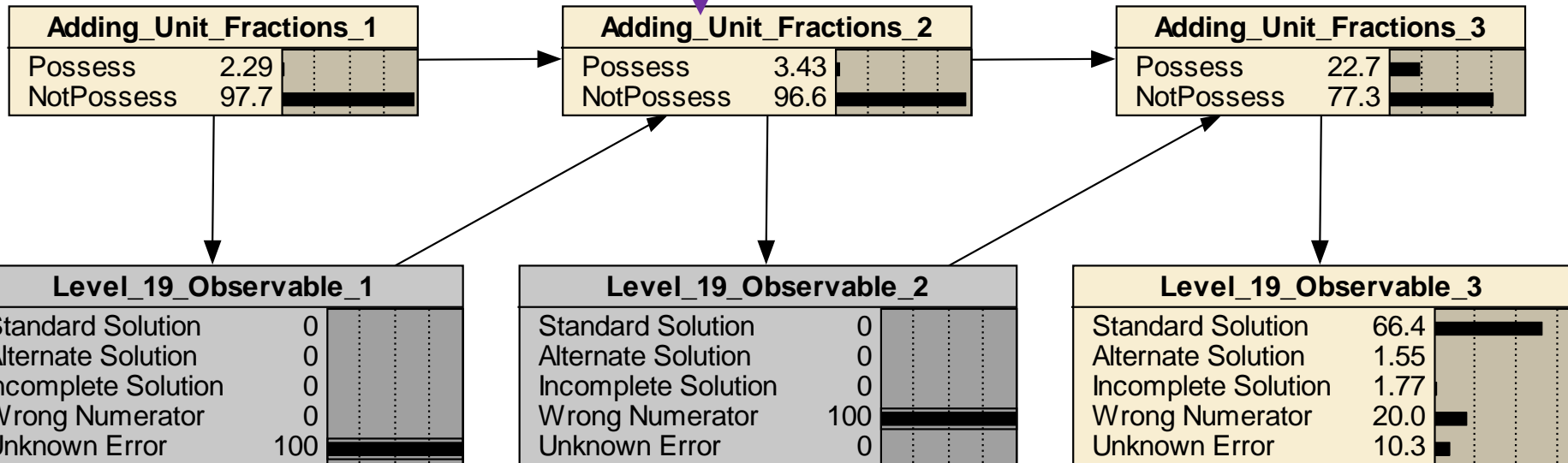


Observe a wrong numerator
on the second attempt

Session IVb -- DBNs

Posterior Probability

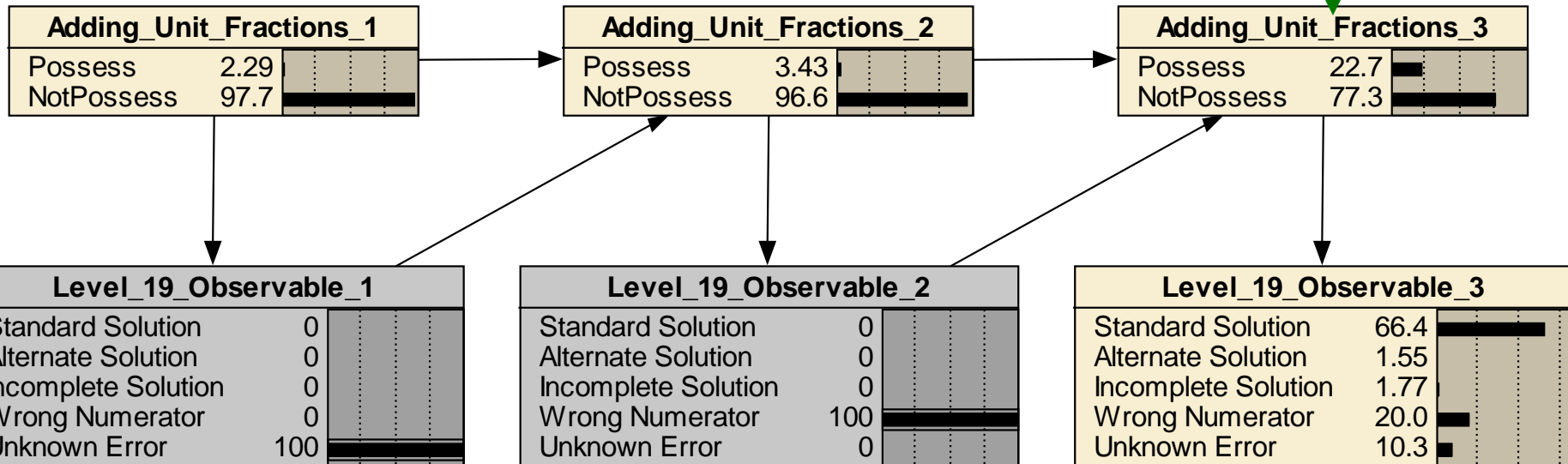
$$p(\text{Possess at time 2} \mid \text{Unkn Err at } t 1, \text{Wrong Num Error at } t 2) = .03$$



Observe a wrong numerator
on the second attempt

Posterior Predictive Probability

$$p(\text{Possess at time 3} \mid \text{Unkn Err at } t 1, \text{Wrong Num Error at } t 2) = .23$$



Observe a wrong numerator
on the second attempt

Posterior Retrodictive Probability

$$p(\text{Possess at time 1} \mid \text{Unkn Err at } t 1, \text{Wrong Num Error at } t 2) = .02$$

Adding_Unit_Fractions_1	
Possess	2.29
NotPossess	97.7

Adding_Unit_Fractions_2	
Possess	3.43
NotPossess	96.6

Adding_Unit_Fractions_3	
Possess	22.7
NotPossess	77.3

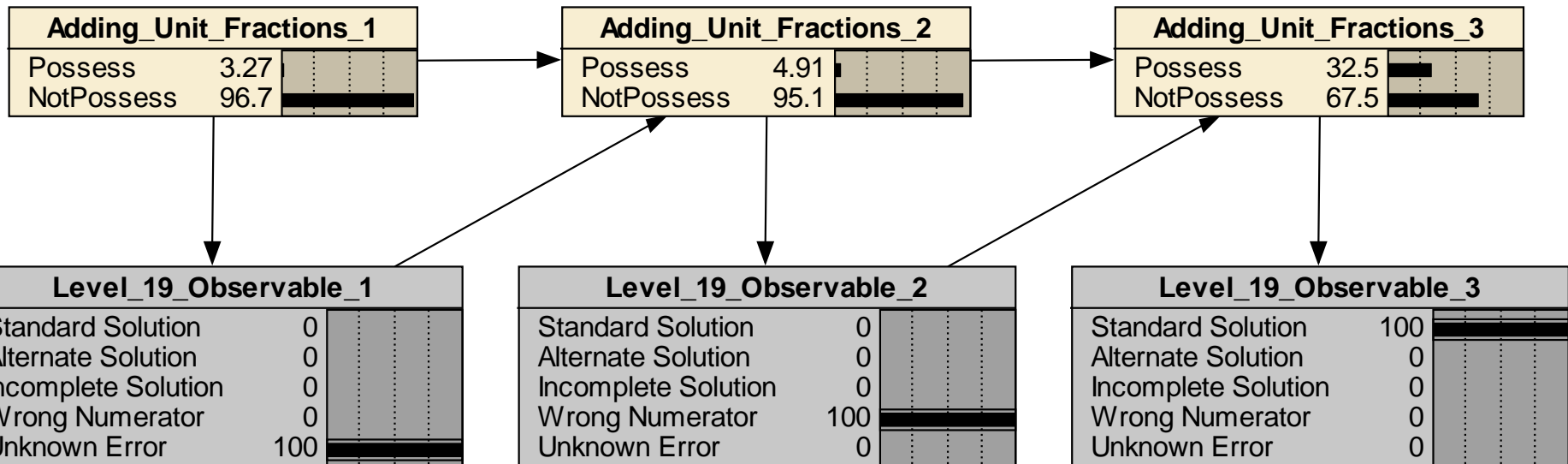
Level_19_Observable_1	
Standard Solution	0
Alternate Solution	0
Incomplete Solution	0
Wrong Numerator	0
Unknown Error	100

Level_19_Observable_2	
Standard Solution	0
Alternate Solution	0
Incomplete Solution	0
Wrong Numerator	100
Unknown Error	0

Level_19_Observable_3	
Standard Solution	66.4
Alternate Solution	1.55
Incomplete Solution	1.77
Wrong Numerator	20.0
Unknown Error	10.3

Observe a wrong numerator
on the second attempt

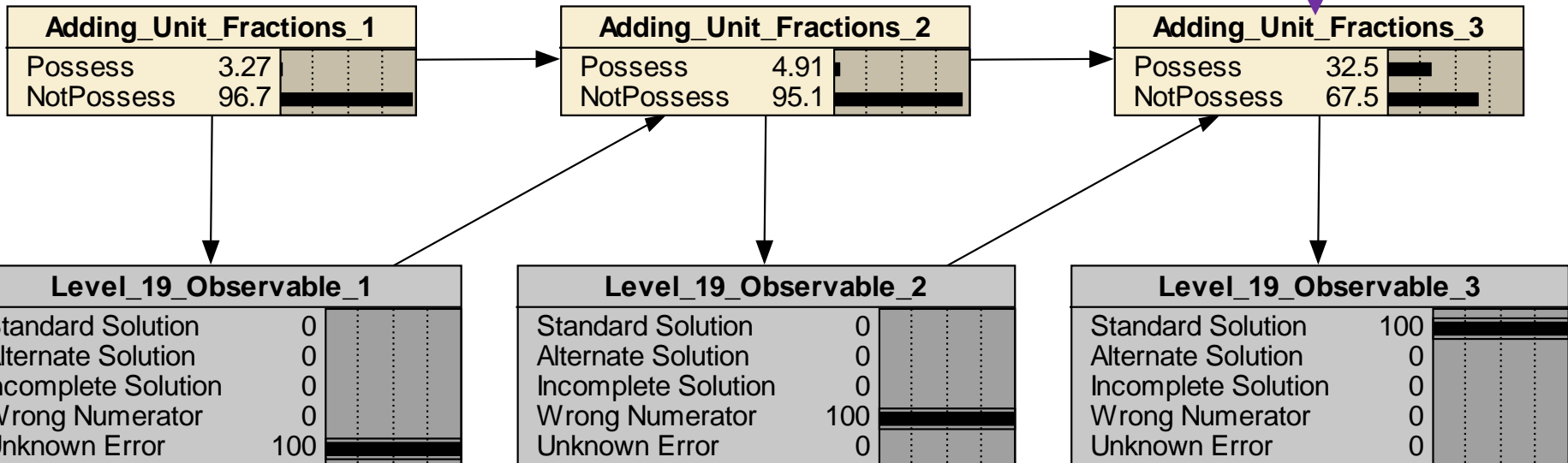
3rd Observation



Observe a standard solution
on the third attempt

Posterior Probability

$$p(\text{Possess at time 3} \mid \text{Unkn Err, Wrong Num Err, Stand Sol}) = .33$$



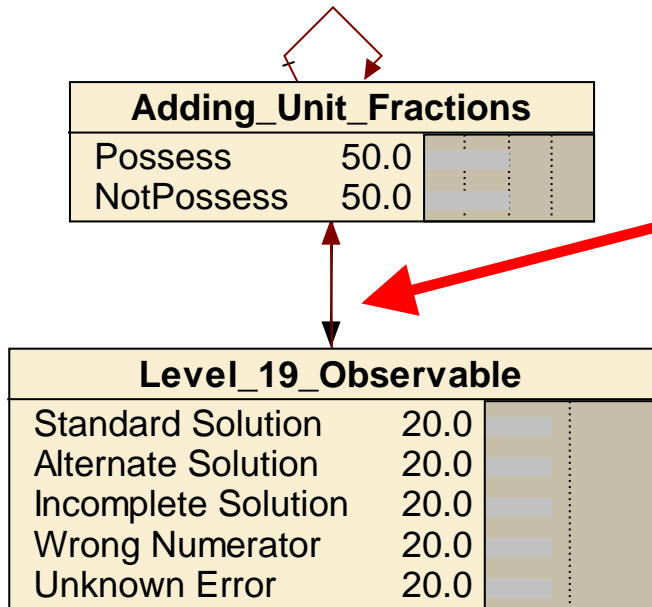
Observe a standard solution
on the third attempt

Netica Files:

Save Patch Level 19 Simplified Dynamic.neta

*Save Patch Level 19 Simplified Dynamic Expanded 3
Time Points.neta*

Save Patch Level 19 Simplified Dynamic.neta

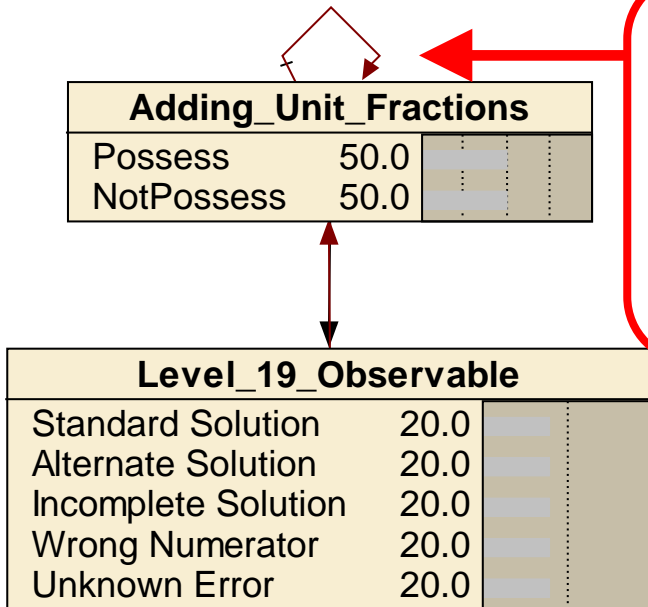


Link from Adding_Unit_Fractions to Level_19_Observable indicates observable at any time depends on proficiency at that time

Observable for Level 19 at Time t

<i>Adding Unit Fractions at Time t</i>	Standard Solution	Alternate Solution	Incomplete Solution	Wrong Numerator Error	Unknown Error
Possess	1	1	1	1	1
Not Possess	.38	.17	.19	.20	.09

Save Patch Level 19 Simplified Dynamic.neta

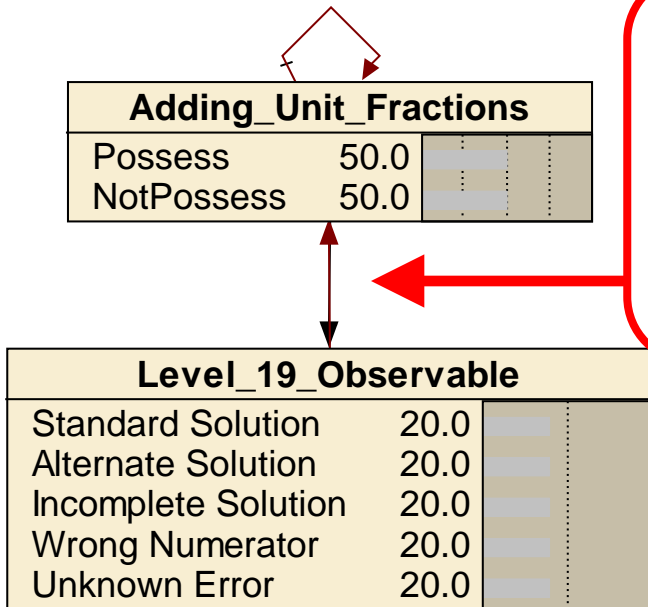


Link from Adding_Unit_Fractions to itself indicates that Adding_Unit_Fractions at any time depends on Adding_Unit_Fractions in the past

Input Delay = 1 means lag of 1

The screenshot shows the configuration window for "Adding_Unit_Fractions (node of Save_Patch_Level_19_Simplified)". The "Name" field is "Adding_Unit_Fractions" and the "Title" field is empty. The "Nature" is set to "Discrete". The "State" is "Possess" with "New" and "Delete" buttons. The "Value" field is empty. The "Input Delay" is set to 1, and the "Parent" is "Adding_Unit_". The "Input Delay" field and its dropdown are highlighted with a red box. On the right side, there are buttons for "OK", "Apply", "Reset", "Close", "Table", and "Help".

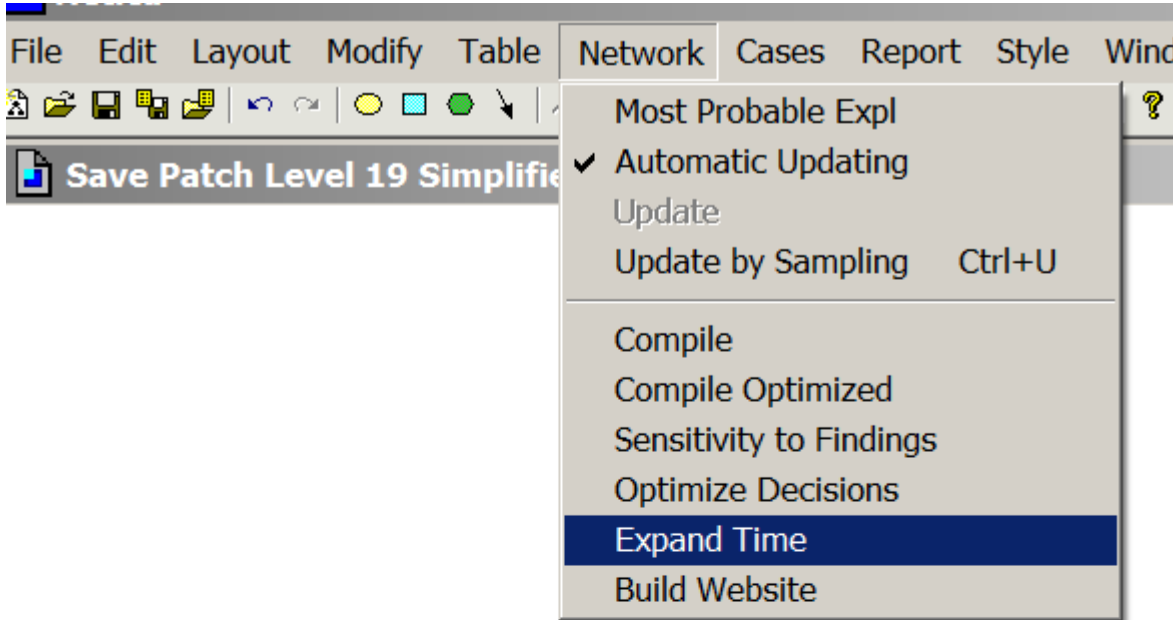
Save Patch Level 19 Simplified Dynamic.neta



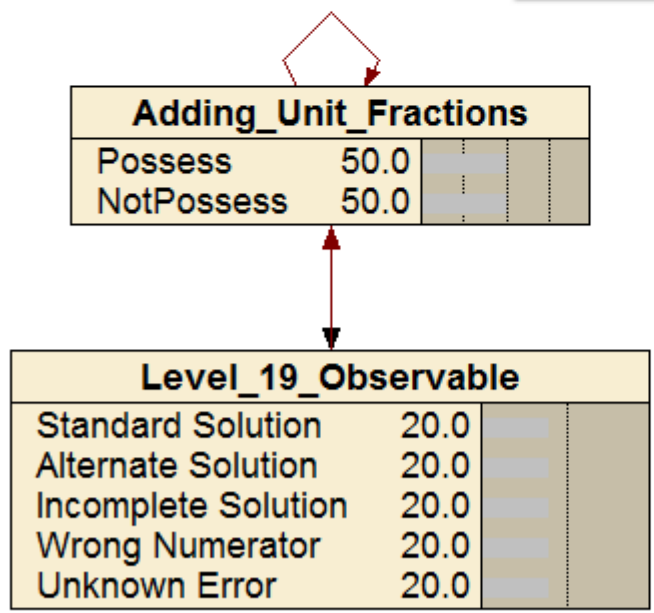
Link from Level_19_Observable to Adding_Unit_Fractions to indicates future proficiency at any time depends on current observable

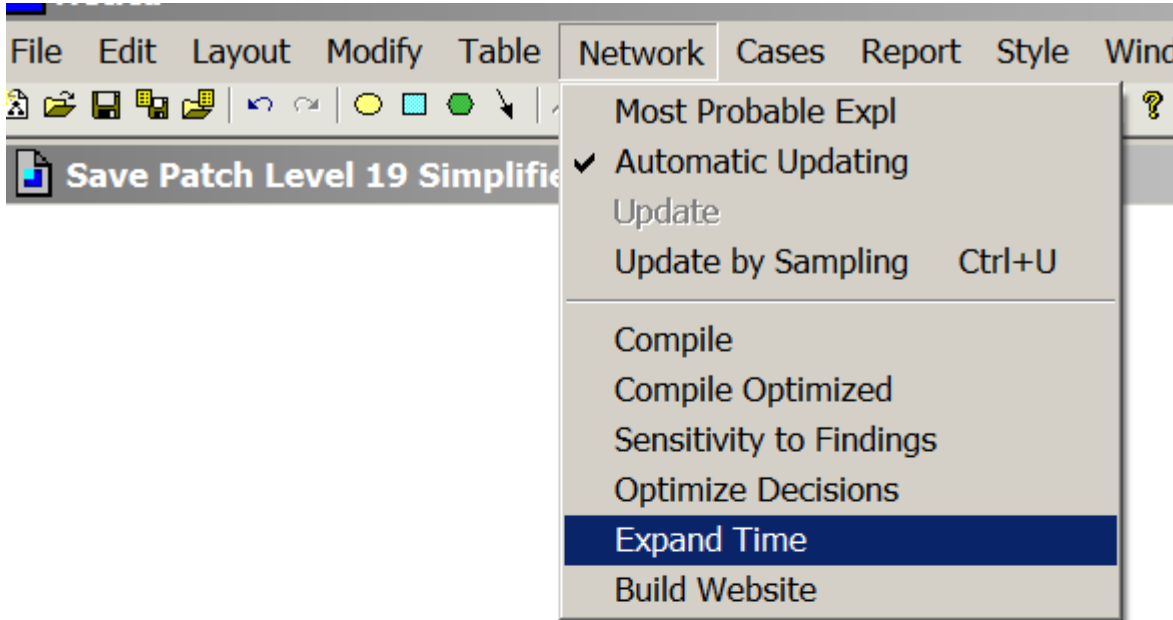
The screenshot shows the configuration window for the 'Adding_Unit_Fractions' node. The window title is 'Adding_Unit_Fractions (node of Save_Patch_Level_19_Simplified)'. The 'Name' field is 'Adding_Unit_Fractions' and the 'Title' field is empty. The 'Nature' is set to 'Discrete'. The 'State' is 'Possess' and the 'Value' is empty. The 'Input Delay' is set to 1 and the 'Parent' is 'Level_19_Obs_'. The window also contains buttons for 'New', 'Delete', 'OK', 'Apply', 'Reset', 'Close', 'Table', and 'Help'.

Input Delay = 1 means lag of 1



ified Dynamic.neta





ified Dynamic.neta

How many additional time slices?

Adding_Unit_Fractions

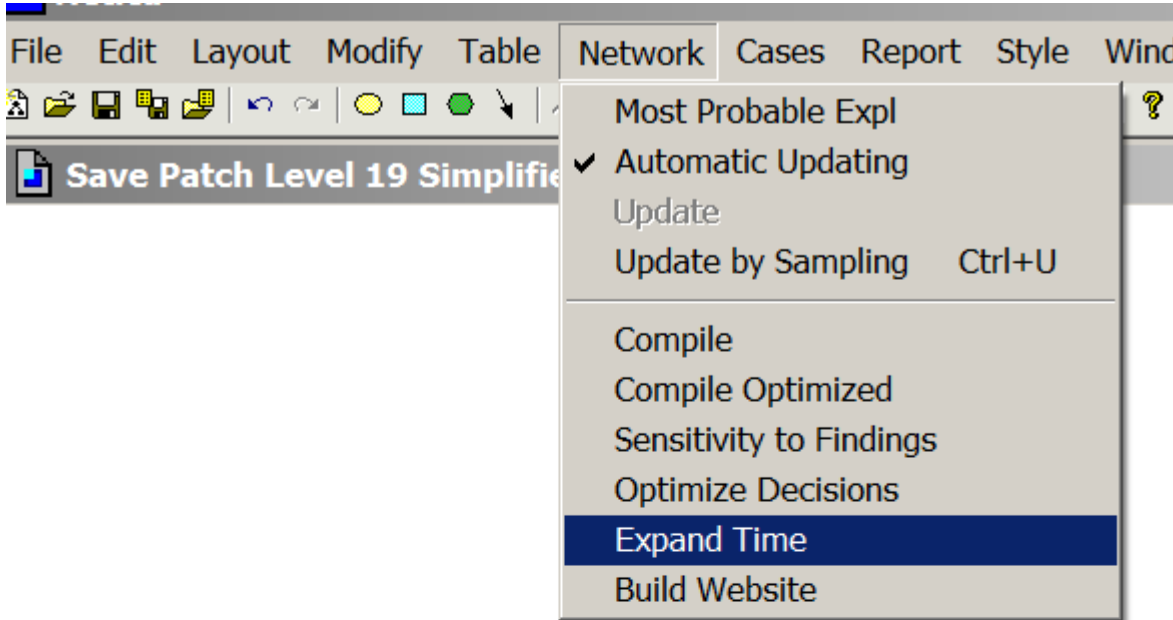
Possess	50.0		
NotPossess	50.0		

Level_19_Observable

Standard Solution	20.0		
Alternate Solution	20.0		
Incomplete Solution	20.0		
Wrong Numerator	20.0		
Unknown Error	20.0		

Enter amount of time for expansion:
(if link delays are 1, this is the number of time steps)

OK Revert Cancel



ified Dynamic.neta

When to start?

Adding_Unit_Fractions

Possess	50.0		
NotPossess	50.0		

Level_19_Observable

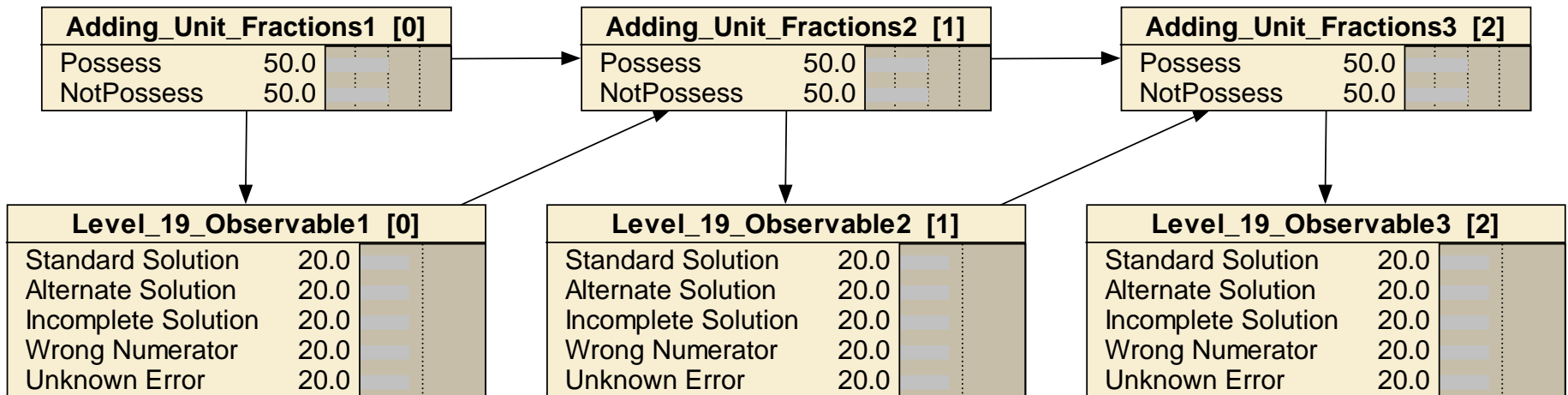
Standard Solution	20.0		
Alternate Solution	20.0		
Incomplete Solution	20.0		
Wrong Numerator	20.0		
Unknown Error	20.0		

Enter burn-in time:

OK Revert Cancel

Save Patch Level 19 Simplified Dynamic.neta

Must edit the table for the first time point



Save Patch Level 19 Simplified Dynamic.neta

Must edit the table for the first time point

The image shows a Bayesian network editor interface. On the left, there are three tables representing nodes in the network:

- Adding_Unit_Fractions1 [0]**:

Possess	50.0
NotPossess	50.0
- Level_19_Observable1 [0]**:

Standard Solution	20.0
Alternate Solution	20.0
Incomplete Solution	20.0
Wrong Numerator	20.0
Unknown Error	20.0
- Adding_Unit_Fractions1 [1]**:

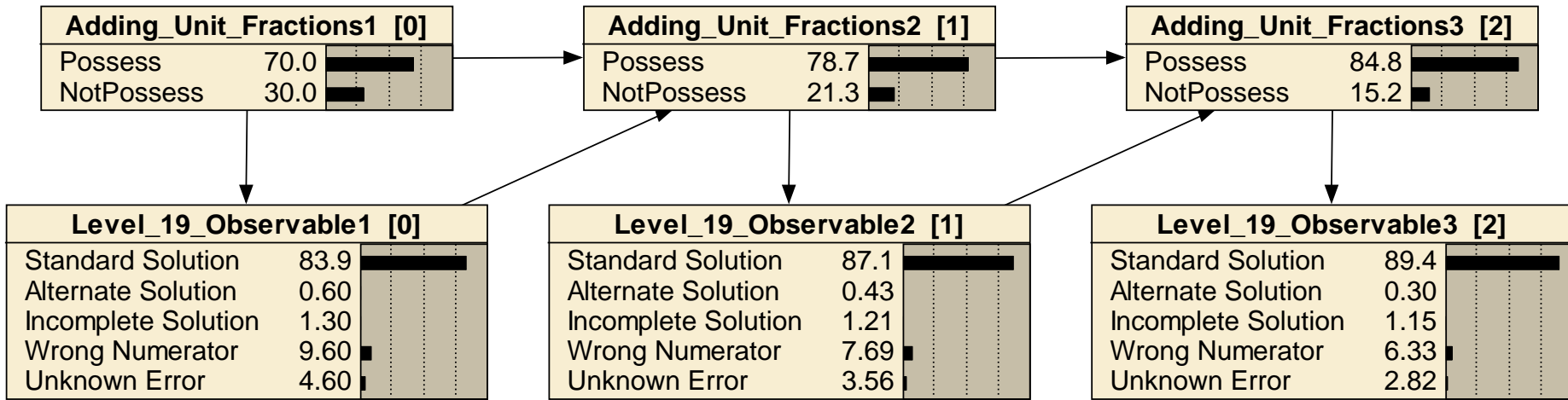
Possess	50.0
NotPossess	50.0

Arrows indicate dependencies: 'Adding_Unit_Fractions1 [0]' points to 'Adding_Unit_Fractions1 [1]' and 'Level_19_Observable1 [0]'. A dialog box titled 'Adding_Unit_Fractions1 Table (in Bayes ...)' is open, showing the node 'Adding_Unit_F_' and a table for editing probabilities:

Possess	NotPossess...
.7	.3

The dialog box also contains buttons for 'Apply', 'OK', 'Chance', 'Probabi_', 'Reset', and 'Close'.

Save Patch Level 19 Simplified Dynamic.neta



See

‘Save Patch Level 19 Simplified Dynamic Expanded 3 Time Points.neta’