<table>
<thead>
<tr>
<th>SESSION</th>
<th>TOPIC</th>
<th>PRESENTERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 1</td>
<td>Evidence Centered Design</td>
<td>Diego Zapata</td>
</tr>
<tr>
<td></td>
<td>Bayesian Networks</td>
<td></td>
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<tr>
<td>Session 2</td>
<td>Bayes Net Applications</td>
<td>Duanli Yan &amp;</td>
</tr>
<tr>
<td></td>
<td>ACED: ECD in Action</td>
<td>Russell Almond</td>
</tr>
<tr>
<td>Session 3</td>
<td>Refining Bayes Nets with Data</td>
<td>Russell Almond</td>
</tr>
<tr>
<td>Session 4</td>
<td>Bayes Nets with R</td>
<td>Duanli Yan &amp;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Russell Almond</td>
</tr>
</tbody>
</table>
1. Discrete Item Response Theory (IRT)

- Proficiency Model
- Task/Evidence Models
- Assembly Model
- Some Numbers
IRT Proficiency Model

• There is one proficiency variable, \( \theta \). (Sometimes called an “ability parameter”, but we reserve the term parameter for quantities which are not person specific.)

• \( \theta \) takes on values \{-2, -1, 0, 1, 2\} with prior probabilities of \((0.1, 0.2, 0.4, 0.2, 0.1)\) (Triangular distribution).

• Observable outcome variables are all independent given \( \theta \)

• Goal is to draw inferences about \( \theta \)
  – Rank order students by \( \theta \)
  – Classify students according to \( \theta \) above or below a cut point
IRT Task/Evidence Model

- Tasks yield an work product which can be unambiguously scored right/wrong.
- Each task has a single observable outcome variable.
- *Tasks* are often called *items*, although the common usage often blurs the distinction between the presentation of the item and the outcome variable.
IRT (Rasch) Evidence Model

- Let $X_j$ be observable outcome variable from Task $j$

- $P(X_j = \text{right} \mid \theta, \beta_j) = \frac{1}{1 + e^{-(\theta - \beta_j)}}$

  $\beta_j$ is the difficulty of the item.

- Can crank through the formula for each of the five values of $\theta$ to get values for Conditional Probability Tables (CPT)
IRT Assembly Model

• 5 items
• Increasing difficulty:
  \( \beta \in \{-1.5, -0.75, 0, 0.75, 1.5\} \).
• Adaptive presentation of items
## Conditional Probability Tables

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>Prior</th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
<th>Item 4</th>
<th>Item 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0.1</td>
<td>0.3775</td>
<td>0.2227</td>
<td>0.1192</td>
<td>0.0601</td>
<td>0.0293</td>
</tr>
<tr>
<td>-1</td>
<td>0.2</td>
<td>0.6225</td>
<td>0.4378</td>
<td>0.2689</td>
<td>0.1480</td>
<td>0.0759</td>
</tr>
<tr>
<td>0</td>
<td>0.4</td>
<td>0.8176</td>
<td>0.6792</td>
<td>0.5000</td>
<td>0.3208</td>
<td>0.1824</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
<td>0.9241</td>
<td>0.8520</td>
<td>0.7311</td>
<td>0.5622</td>
<td>0.3775</td>
</tr>
<tr>
<td>2</td>
<td>0.1</td>
<td>0.9707</td>
<td>0.9399</td>
<td>0.8088</td>
<td>0.7773</td>
<td>0.6225</td>
</tr>
</tbody>
</table>
Problems Set 1

1. Assume $\theta=1$, what is expected score (sum $X_j$)
2. Calculate $P(\theta | X_1=\text{right})$, $E(\theta | X_1=\text{right})$
3. Calculate $P(\theta | X_5=\text{right})$, $E(\theta | X_5=\text{right})$
4. Score three students who have the following observable patterns (Tasks 1--5):
   1,1,1,0,0
   1,0,0,1,1
   1,1,1,0,1
5. Suppose we have observed for a given student $X_2=\text{right}$ and $X_3=\text{right}$, what is the next best item to present (hint, look for expected probabilities closest to .5,.5)
6. Same thing, with $X_2=\text{right}$ and $X_3=\text{wrong}$
7. Same thing, with $X_2=\text{wrong}$ and $X_3=\text{wrong}$
2. “Context” effect -- Testlets

- Standard assumption of conditional independence of observable variables given Proficiency Variables
- Violation
  - Shared stimulus
  - Context
  - Special knowledge
  - Shared Work Product
  - Sequential dependencies
  - Scoring Dependencies (Multi-step problem)
- Testlets (Wainer & Kiely, 1987)
- Violation results in overestimating the evidential value of observables for Proficiency Variables
“Context” effect -- Variables

- Context variable – A parent variable introduced to handle conditional dependence among observables (testlet)
  - Consistent with Stout’s (1987) ‘essential n-dimensionality’
  - Wang, Bradlow & Wainer (2001) SCORIGHT program for IRT
  - Patz & Junker (1999) model for multiple ratings
“Context” effect -- example

• Suppose that Items 3 and 4 share common presentation material

• Example: a word problem about “Yacht racing” might use nautical jargon like “leeward” and “tacking”

• People familiar with the content area would have an advantage over people unfamiliar with the content area.

• Would never use this example in practice because of DIF (Differential Item Functioning)
Adding a context variable

• Group Items 3 and 4 into a single task with two observed outcome variables
• Add a person-specific, task-specific latent variable called “context” with values familiar and unfamiliar
• Estimates of $\theta$ will “integrate out” the context effect
• Can use as a mathematical trick to force dependencies between observables.
IRT Model with Context Variable
Problem Set 2

• Compare the following quantities in the context and no context models:
  1. $P(X_2)$, $P(X_3)$, $P(X_4)$
  2. $P(\theta|X_2=\text{right})$, $P(\theta|X_3=\text{right})$
  3. $P(X_4|X_2=\text{right})$, $P(X_4|X_3=\text{right})$
  4. $P(\theta|X_3=\text{wrong}, X_4=\text{wrong})$, $P(\theta|X_3=\text{right}, X_4=\text{wrong})$
  5. $P(\theta|X_3=\text{wrong}, X_4=\text{right})$, $P(\theta|X_3=\text{right}, X_4=\text{right})$
Context Effect Postscript

• If Context effect is generally construct-irrelevant variance, if correlated with group membership this is bad (DIF)

• When calibrating using 2PL IRT model, can get similar joint distribution for $\theta$, $X_3$, and $X_4$ by decreasing the discrimination parameter
3. Combination Models

Consider a task which requires two Proficiencies: Three different ways to combine those proficiencies:

- **Compensatory**: More of Proficiency 1 compensates for less of Proficiency 2. Combination rule is \( \text{sum} \).

- **Conjunctive**: Both proficiencies are needed to solve the problem. Combination rule is \( \text{minimum} \).

- **Disjunctive**: Two proficiencies represent alternative solution paths to the problem. Combination rule is \( \text{maximum} \).
Combination Model Graphs
Common Setup for All Three Models

• There are two parent nodes, and both parents are conditionally independent of each other. The difference among the three models lies in the third term below:

\[ P(P_1, P_2, X) = P(P_1) \cdot P(P_2) \cdot P(X | P_1, P_2) \]

• The priors for the parent nodes are the same for the three models with 0.3333 of probability at each of the H, M, and L states.

• The initial marginal probability for X is the same for the three models (50/50).
Conditional Probability Tables

This table contains the conditional probabilities for the parent nodes (P1 and P2) and the combination model for the three models.

Table 3 – Part 2
Conditional Problems for Compensatory, Conjunctive, and Disjunctive

<table>
<thead>
<tr>
<th>P1</th>
<th>P2</th>
<th>Compensatory “Right”</th>
<th>Conjunctive “Right”</th>
<th>Disjunctive “Right”</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>H</td>
<td>0.9</td>
<td>0.9</td>
<td>0.7</td>
</tr>
<tr>
<td>H</td>
<td>M</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>H</td>
<td>L</td>
<td>0.5</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>M</td>
<td>H</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>M</td>
<td>M</td>
<td>0.5</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td>M</td>
<td>L</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>L</td>
<td>H</td>
<td>0.5</td>
<td>0.3</td>
<td>0.7</td>
</tr>
<tr>
<td>L</td>
<td>M</td>
<td>0.3</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>L</td>
<td>L</td>
<td>0.1</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Problem Set 3

1. Verify that \( P(P_1), P(P_2), \) and \( P(Obs) \) are the same for all three models. (\( Obs \) represents either the node \textit{Compensatory}, \textit{Conjunctive}, or \textit{Disjunctive} )

2. Assume \( Obs=\text{right} \), Calculate \( P(P_1) \) and \( P(P_2) \) for all three models.

3. Assume \( Obs=\text{wrong} \), Calculate \( P(P_1) \) and \( P(P_2) \) for all three models.

4. Assume \( Obs=\text{right} \), and \( P_1 = H \). Calculate \( P(P_2) \) for all three models.

5. Assume \( Obs=\text{right} \), and \( P_1 = M \). Calculate \( P(P_2) \) for all three models.

6. Assume \( Obs=\text{right} \), and \( P_1 = L \). Calculate \( P(P_2) \) for all three models.

7. Explain the differences
Activity 3

• Go back to the Driver’s License Exam you built in Session I and add some numbers

• Now put in some observed outcomes
  – How did the probabilities change?
  – Is that about what you expected?
ACED Background

- ACED (Adaptive Content with Evidence-based Diagnosis)
- Val Shute (PD), Aurora Graf, Jody Underwood, Eric Hansen, Peggy Redman, Russell Almond, Larry Casey, Waverly Hester, Steve Landau, Diego Zapata
- Domain: Middle School Math, Sequences
- Project Goals:
  - Adaptive Task Selection
  - Diagnostic Feedback
  - Accessibility
Valid Assessment. Based on evidence-centered design (ECD).

Adaptive Sequencing. Tasks presented in line with an adaptive algorithm.

Diagnostic Feedback. Feedback is immediate and addresses common errors and misconceptions.

Aligned. Assessments aligned with (a) state and national standards and (b) curricula in current textbooks.
ACED Proficiency Model

Sequences as Patterns

Arithmetic
- Extend
- Table
- Pictorial
- InduceRules
- Examples
- Common Difference
- Algebra Rule
- VerbalRule
- Recursive
- Explicit

Other Recursive
- Extend
- Table
- Pictorial
- InduceRules
- Algebra Rule
- VerbalRule
- Recursive
- Explicit

Geometric
- Extend
- Table
- Pictorial
- InduceRules
- Common Ratio
- Algebra Rule
- VerbalRule
- Recursive
- Explicit

Model
Examples
Katie is a biochemist. During her last trip to the Amazon rainforest, she brought back some leaves from an exotic plant. She extracted a substance from those leaves that had some amazing properties. One drop of the substance on a given cell produced a doubling of the cell, along with a smaller bonus cell (see Stages 1 and 2, below). The same pattern was found in consecutive trials (see Stages 3-4).

She made a table of her findings. Your task is to figure out how many blue, red, and total cells would be present in the 8th sequence. Complete the table by filling in the values for A, B, and C (where N = 8).

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Blue</th>
<th>Red</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>N</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
</tbody>
</table>
ACED Design/Build Process

- Identify Proficiency variables
- Structure Proficiency Model
- Elicit Proficiency Model Parameters
- Construct Tasks to target proficiencies at Low/Medium/High difficulty
- Build Evidence Models based on difficulty/Q-Matrix
Parameterization of Network

• Proficiency Model:
  – Based on Regression model of child given parent
  – SME provided correlation and intercept
  – SME has low confidence in numeric values

• Evidence Model Fragment
  – Tasks Scored Right/Wrong
  – Based on IRT model
  – High/Medium/Low corresponds to $\theta = +1/0/-1$
  – Easy/Medium/Hard corresponds to difficulty $-1/0/+1$
  – Discrimination of 1
  – Used Q-Matrix to determine which node is parent
PM-EM Algorithm for Scoring

• Master Bayes net with just proficiency model (PM)
• Database of Bayes net fragments corresponding to evidence models (EMs), indexed by task ID
• To score a task:
  – Find EM fragment corresponding to task
  – Join EM fragment to PM
  – Enter Evidence
  – Absorb evidence from EM fragment into network
  – Detach EM fragment
An Example

- Five proficiency variables
- Three tasks, with observables \( \{X_{11}\}, \{X_{21}, X_{22}, X_{23}\}, \{X_{31}\} \).
Q: Which observables depend on which proficiency variables?
A: See the Q-matrix (Fischer, Tatsuoka).

<table>
<thead>
<tr>
<th></th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$X_{23}$</th>
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</thead>
<tbody>
<tr>
<td>$X_{11}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>--</td>
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<tr>
<td>$X_{21}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$X_{22}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$X_{23}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>N/A</td>
</tr>
<tr>
<td>$X_{31}$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>--</td>
</tr>
</tbody>
</table>
Proficiency Model / Evidence Model Split

- Full Bayes net for proficiency model and observables for all tasks can be decomposed into fragments.
  - Proficiency model fragment(s) (PMFs) contain proficiency variables.
  - An evidence model fragment (EMF) for each task.
  - EMF contains observables for that task and all proficiency variables that are parents of any of them.
- Presumes observables are conditionally independent between tasks, but can be dependent within tasks.
- Allows for adaptively selecting tasks, docking EMF to PMF, and updating PMF on the fly.
On the way to PMF and EMFs...

Proficiency variables

Observables and proficiency variable parents for the tasks
Marry parents, drop directions, and triangulate (in PMF, with respect to all tasks)
Footprints of tasks in proficiency model (figure out from rows in Q-matrix)
Result:

- Each EMF implies a join tree for Bayes net propagation.
  - Initial distributions for proficiency variables are uniform.
- The footprint of the PM in the EMF is a clique intersection between that EMF and the PMF.
- Can “dock” EMFs with PMF one-at-a-time, to …
  - absorb evidence from values of observables to that task as updated probabilities for proficiency variables, and
  - predict responses in new tasks, to evaluate potential evidentiary value of administering it.
Docking evidence model fragments

PMF

April 2019

2019 NCME Tutorial: Bayesian Networks in Educational Assessment
## Scoring Exercise

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Task Name</th>
<th>Proficiency Variable</th>
<th>Difficulty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wrong</td>
<td>tCommonRatio1a.xml</td>
<td>CommonRatio</td>
<td>Easy</td>
</tr>
<tr>
<td>Right</td>
<td>tCommonRatio2b.xml</td>
<td>CommonRatio</td>
<td>Medium</td>
</tr>
<tr>
<td>Wrong</td>
<td>tCommonRatio3b.xml</td>
<td>CommonRatio</td>
<td>Hard</td>
</tr>
<tr>
<td>Wrong</td>
<td>tExplicitGeometric1a.xml</td>
<td>ExplicitGeometric</td>
<td>Easy</td>
</tr>
<tr>
<td>Right</td>
<td>tExplicitGeometric2a.xml</td>
<td>ExplicitGeometric</td>
<td>Medium</td>
</tr>
<tr>
<td>Wrong</td>
<td>tExplicitGeometric3b.xml</td>
<td>ExplicitGeometric</td>
<td>Hard</td>
</tr>
<tr>
<td>Wrong</td>
<td>tRecursiveRuleGeometric1a.xml</td>
<td>RecursiveRuleGeometric</td>
<td>Easy</td>
</tr>
<tr>
<td>Wrong</td>
<td>tRecursiveRuleGeometric2b.xml</td>
<td>RecursiveRuleGeometric</td>
<td>Medium</td>
</tr>
<tr>
<td>Wrong</td>
<td>tRecursiveRuleGeometric3a.xml</td>
<td>RecursiveRuleGeometric</td>
<td>Hard</td>
</tr>
<tr>
<td>Right</td>
<td>tTableExtendGeometric1a.xml</td>
<td>TableGeometric</td>
<td>Easy</td>
</tr>
<tr>
<td>Right</td>
<td>tTableExtendGeometric2b.xml</td>
<td>TableGeometric</td>
<td>Medium</td>
</tr>
<tr>
<td>Right</td>
<td>tTableExtendGeometric3a.xml</td>
<td>TableGeometric</td>
<td>Hard</td>
</tr>
<tr>
<td>Wrong</td>
<td>tVerbalRuleExtendModelGeometric1a.xml</td>
<td>VerbalRuleGeometric</td>
<td>Easy</td>
</tr>
<tr>
<td>Wrong</td>
<td>tVerbalRuleExtendModelGeometric1b.xml</td>
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<td>Easy</td>
</tr>
<tr>
<td>Right</td>
<td>tVerbalRuleExtendModelGeometric2a.xml</td>
<td>VerbalRuleGeometric</td>
<td>Medium</td>
</tr>
<tr>
<td>Wrong</td>
<td>tVisualExtendGeometric1a.xml</td>
<td>VisualGeometric</td>
<td>Easy</td>
</tr>
<tr>
<td>Wrong</td>
<td>tVisualExtendGeometric2a.xml</td>
<td>VisualGeometric</td>
<td>Medium</td>
</tr>
<tr>
<td>Wrong</td>
<td>tVisualExtendGeometric3a.xml</td>
<td>VisualGeometric</td>
<td>Hard</td>
</tr>
</tbody>
</table>
Weight of Evidence

• Good (1985)
• $H$ is binary hypothesis, e.g., *Proficiency > Medium*
• $E$ is evidence for hypothesis
• Weight of Evidence (WOE) is

\[ W(H : E) = \log \frac{\Pr(E|H)}{\Pr(E|\overline{H})} = \log \frac{\Pr(H|E)}{\Pr(H|\overline{E})} - \log \frac{\Pr(H)}{\Pr(\overline{H})} \]
Properties of WOE

• “Centibans” (log base 10, multiply by 100)
• Positive for evidence supporting hypothesis, negative for evidence refuting hypothesis
• Movement in tails of distribution as important as movement near center
• Bayes theorem using log odds
Conditional Weight of Evidence

• Can define Conditional Weight of Evidence

\[ W(H : E_2|E_1) = \log \frac{\Pr(E_2|H, E_1)}{\Pr(E_2|\overline{H}, E_1)} \]

• Nice Additive properties

\[ W(H : E_1, E_2) = W(H : E_1) + W(H : E_2|E_1) \]

• Order sensitive

• WOE Balance Sheet (Madigan, Mosurski & Almond, 1997)
### Evidence Balance Sheet

#### Tasks Overview

<table>
<thead>
<tr>
<th>Task</th>
<th>Acc</th>
<th>P(Solve Geom Sequences)</th>
<th>WOE for H vs. M, L</th>
</tr>
</thead>
<tbody>
<tr>
<td>SolveGeometricProblems2a</td>
<td>0</td>
<td>H: 0.16, M: 0.26, L: 0.58</td>
<td></td>
</tr>
<tr>
<td>SolveGeometricProblems3a</td>
<td>1</td>
<td>H: 0.35, M: 0.35, L: 0.30</td>
<td></td>
</tr>
<tr>
<td>SolveGeometricProblems3b</td>
<td>1</td>
<td>H: 0.64, M: 0.29, L: 0.07</td>
<td></td>
</tr>
<tr>
<td>SolveGeometricProblems2b</td>
<td>1</td>
<td>H: 0.83, M: 0.16, L: 0.01</td>
<td></td>
</tr>
<tr>
<td>VisualExtendTable2a</td>
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<td>H: 0.89, M: 0.10, L: 0.01</td>
<td></td>
</tr>
<tr>
<td>SolveGeometricProblems1a</td>
<td>0</td>
<td>H: 0.78, M: 0.21, L: 0.01</td>
<td></td>
</tr>
<tr>
<td>SolveGeometricProblems1b</td>
<td>1</td>
<td>H: 0.82, M: 0.18, L: 0.00</td>
<td></td>
</tr>
<tr>
<td>VisualExtendVerbalRule2a</td>
<td>1</td>
<td>H: 0.85, M: 0.15, L: 0.00</td>
<td></td>
</tr>
<tr>
<td>ModelExtendTableGeometric3a</td>
<td>1</td>
<td>H: 0.90, M: 0.10, L: 0.00</td>
<td></td>
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<tr>
<td>ExamplesGeometric2a</td>
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<td>H: 0.87, M: 0.13, L: 0.00</td>
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</tr>
<tr>
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<td>H: 0.91, M: 0.09, L: 0.00</td>
<td></td>
</tr>
<tr>
<td>VerbalRuleModelGeometric3a</td>
<td>1</td>
<td>H: 0.95, M: 0.05, L: 0.00</td>
<td></td>
</tr>
</tbody>
</table>

#### Note

- **Acc** represents the accuracy of the task completion.
- **P(Solve Geom Sequences)** shows the probability of solving geometric sequences.
- **WOE for H vs. M, L** indicates the Weight of Evidence for levels H, M, and L compared to each other.
Expected Weight of Evidence

When choosing next “test” (task/item) look at expected value of WOE where expectation is taken wrt $P(E|H)$.

$$EW(H : E) = \sum_{j=1}^{n} W(H : e_j)Pr(e_j | H)$$

where $\{e_j, j = 1, ..., n\}$ represent the possible results.
Calculating EWOE

Madigan and Almond (1996)

1. Instantiate Hypothesis = True (may need to use virtual evidence if hypothesis is compound)
2. Calculate $P(E_i | H)$ for each candidate item
3. Instantiate Hypothesis = False
4. Calculate $P(E_i | \overline{H})$ for each candidate item
Related Measures

- Value of Information

$$\text{VoI}(T) = E_T \left[ \max_d E_S u(d, S) - \max_d E_{S|T} u(d, S) \right]$$

- $S$ is proficiency state
- $d$ is decision
- $u$ is utility
Related Measures (2)

• Mutual Information

• Extends to non-binary hypothesis nodes

\[ \sum_{x,y} P(x, y) \log \frac{P(x, y)}{P(x)P(y)} \]

• Kullback-Liebler distance between joint distribution and independence

\[ \sum_{x} P(x) \sum_{y} P(y|x) \log \frac{P(y|x)}{P(y)} \]
Task Selection Exercise 1

- Use ACEDMotif1.dne
  - Easy, Medium, and Hard tasks for Common Ratio and Visual Geometric
- Use Hypothesis SolveGeometricProblems > Medium
- Calculate EWOB for six observables
- Assume candidate gets first item right and repeat
- Next assume candidate gets first item wrong and repeat
- Repeat exercise using hypothesis SolveGeometricProblems > Low
Task Selection Exercise 2

- Use Network ACEDMotif2.dne
- Select the SolveGeometricProblems node
- Run the program Network>Sensitivity to Findings
- This will list the Mutual information for all nodes

- Select the observable with the highest mutual information as the first task
- Use this to process a person who gets every task right
- Use this to process a person who gets every task wrong
ACED Evaluation

- Middle School Students
- Did not normally study geometric series
- Four conditions:
  - Elaborated Feedback/Adaptive (E/A; n=71)
  - Simple Feedback/Adaptive (S/A; n=75)
  - Elaborated Feedback/Linear (E/L; n=67)
  - Control (no instruction; n=55)
- Students given all 61 geometric items
- Also given pretest/posttest (25 items each)
ACED Scores

- For Each Proficiency Variable
  - Marginal Distribution
  - Modal Classification
  - EAP Score (High=1, Low=-1)
ACED Reliability

- Calculated with Split Halves (ECD design)
- Correlation of EAP score with posttest is 0.65 (close to reliability of posttest)
- Even with pretest forced into the equation, EAP score accounted for 17% unique variance
- Reliability of modal classifications was worse

<table>
<thead>
<tr>
<th>Proficiency (EAP)</th>
<th>Reliability</th>
</tr>
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<tbody>
<tr>
<td>Solve Geometric Sequences (SGS)</td>
<td>0.88</td>
</tr>
<tr>
<td>Find Common Ratio</td>
<td>0.90</td>
</tr>
<tr>
<td>Generate Examples</td>
<td>0.92</td>
</tr>
<tr>
<td>Extend Sequence</td>
<td>0.86</td>
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<tr>
<td>Model Sequence</td>
<td>0.80</td>
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<tr>
<td>Use Table</td>
<td>0.82</td>
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<tr>
<td>Use Pictures</td>
<td>0.82</td>
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<tr>
<td>Induce Rules</td>
<td>0.78</td>
</tr>
<tr>
<td>Number Right</td>
<td>0.88</td>
</tr>
</tbody>
</table>
Effect of Adaptivity

- For adaptive conditions, correlation with posttest seems to hit upper limit by 20 items
- Standard Error of Correlations is large
- Jump in linear case related to sequence of items
Effect of feedback

- E/A showed significant gains
- Others did not
- Learning and assessment reliability!!!!!
Acknowledgements

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• Complete data available at: http://ecd.ralmond.net/ecdwiki/ACED/ACED